A structural approach to the estimation of the implied equity risk premium

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December 2018

Abstract
This article uses a structural contingent claims model based on free cash flows to equity (FCFE) to derive the equity risk premium implicit in S&P500 stocks. This is done at the aggregate level for the period between 1999 and 2017. The results obtained are compared with those that come out from the traditional single-stage FCFE model. Two assumptions regarding long-term corporate growth expectations are made leading to slightly different results. Setting cash flow growth expectations based on 30-year U.S. bond yields the equity risk premium in December 2017 is found to be 4.6%, very close to the minimum value of the series. When a multiple of analysts forecasts on corporate 3 to 5-year earnings growth is used, the equity risk premium is found to be 5.2%, somewhat closer to the average equity risk premium estimated, which is approximatelly 5.9% in both cases. Under both cases the implied equity risk premium is found to be currently on a downward trend. The higher equity risk premium obtained in the second case is justified by the recent decoupling between analysts forecasts and the long-term risk free rate. This can be the result of analysts optimism on future firm performance but can also be related with the current abnormally low level of long-term interest rates. (JEL: G12, G13, G32)

Introduction

What discount rate is implicit in current stock prices? What expectations about a firm's future performance are consistent with its current market capitalization? These are questions equity analysts often try to answer before issuing recommendations on whether to buy or sell a firm stock. With most traditional indicators suggesting that U.S. equity valuation are very high and the S&P500 staying close to its all-time maximum in the longest bull market in its history, answering these questions has become increasingly relevant not only for financial analysts and academics but also for central banks all over the world. As investors long term projections and risk appetite often move with the business cycle,

Acknowledgements: I would like to thank António Antunes, Nuno Alves, Isabel Horta Correia, Diana Bonfim, Luisa Farinha, José Faias, Pedro Moreira and António Santos for their comments. The opinions expressed in this article are those of the author and do not necessarily coincide with those of Banco de Portugal. Any errors and omissions are the sole responsibility of the author.
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implied growth expectations not compatible with economic projections or an implied equity risk premium significantly below its historical average signal that investors are being either too optimistic or have a risk appetite above the one observed on average in the past. Both cases are usually interpreted as early warning indicators and closely monitored by macroprudential authorities. From a monetary policy standpoint, identifying the determinants behind stock market dynamics has also gained importance during the last years. Among others things, central banks are interested in better understanding how unconventional monetary policy transmits to the different asset classes and how monetary policy normalisation might affect the same asset classes in the forthcoming years.

Equity analysts often try to answer the aforementioned questions by reverse engineering discounted cash flow models (DCF). The procedure is simple. In the case of the constant growth free cash flow to equity (FCFE) model, equity value corresponds to the perpetual sum of future expected cash flows available to shareholders discounted at a rate μ_E that takes into account equity risk. Assuming that FCFE grows forever at a constant rate g below the discount rate, the usual perpetuity formula gives us the equity value:

$$E_0 = \frac{FCFE_0}{\mu_E - g}. \tag{1}$$

Assuming a discount rate based on some asset pricing model as the CAPM, analysts can back out implied FCFE growth rates and compare them with their projections. Alternatively, using their growth projections they can back out the implied discount rate and compare it with the outcome from their preferred asset pricing model. This type of exercise is very popular among practioners and there is a great number of academic papers on this (e.g. Gebhardt *et al.* (2001), Easton *et al.* (2009) and Ohlson and Juettner-Nauroth (2005)). However, it has two important weaknesses. First, equity value is very sensitive to changes both on the discount rate and on growth expectations. Unless analysts use good estimates either for growth expectations or the discount rate, they will end up with very bad estimates on the other. Though this clearly hurts the usefulness of this type of exercise, most practioners still consider that this type of exercise provides a framework to think on the main determinants behind equity valuations. Second, there is substantial model risk. In this regard, it is noteworthy that most models used in practice ignore default risk and the effect on equity valuation of leverage dynamics. In a very recent study, Eisdorfer *et al.* (forthcoming) conclude that shareholders option to default accounts for 19% to 36% of equity value depending on whether the firm is a more or least distress quintile on their analysis. Ignoring this option is thus a severe limitation of deterministic equity valuation models.

This paper does an exercise similar to the one just explained. In this case, long-term growth expectations are assumed and the implied equity risk premium is derived. This is done however using a contingent claims model able to take into account default risk, operating leverage and financial leverage. The approach here proposed also benefits from incorporating information on credit default swap spreads (CDS). The exercise in this article is done at the aggregate level using accounting and market data of 205 firms belonging to the S&P500 for the period between 1999 and 2017. Two alternative growth rate assumptions are considered. First, long-term corporate growth expectations are set based on U.S. 30-year bond yields. Second, normalized analysts 3 to 5-year earnings growth forecasts are used. Growth expectations in this second case have a mean value equal to the ones in the first case, but they are better able to capture analysts' optimism in firm fundamentals. Under both cases the implied equity risk premium is found to be on a downward trend, but still at a level above the one observed in the late nineties. The equity risk premium derived using the structural approach proposed in this article is also shown to be more stable than the one that comes out from the application of the traditional single-stage FCFE model.

Related literature and contribution

Contingent claims models, also known as structural models of corporate liabilities, started with Merton (1974). In this model, a firm financed by equity and a single pure discount bond is considered to honour its commitments if the market value of its assets at debt maturity is higher than debt's nominal value. If not, the firm defaults and shareholders receive 0. In Merton's model equity can thus be seen as a call option on the firm assets with strike equal to nominal debt. Empirical applications of this seminal model showed poor results, but its tremendous insights opened the door to a huge list of academic and non-academic papers that tried to relax its initial restrictive assumptions in order to better fit the data.[1] In most of the models that followed Merton (1974), the market value of a firm assets has been seen as an exogenous traded asset. Breaking with this tradition, Goldstein *et al.* (2001) propose a model where the asset value is seen as a fictive non-traded security whose value corresponds to the perpetual sum of all future earnings before interest and taxes (EBIT). The latter is assumed to follow a geometric Brownian motion implying that the underlying asset is lognormally distributed. In this framework contingent contracts such as equity, bonds and options are all interrelated through the same market price of risk. The lognormal EBIT assumption in this model is not compatible however with negative EBIT values, something often observed. In addition, EBIT is an income account and thus its relation with the firm capacity to generate cash

1. Popular industry applications include Moody's EDF, the CreditGrades model from Deutsche Bank, Goldman Sachs, JPMorgan and the RiskMetrics Group and Credit Suisse CUSP model.

flow is not direct. The model presented in this article overcomes these issues by defining the state variable as the sum of the cash flow from operating and investment activities, interest expenses and any costs termed fixed. This aggregate is seldom negative and thus more suitable to be modelled as a geometric Brownian motion. Adding up non-financial fixed costs such as selling, general and administrative expenses (SG&A) allows us to consider operating leverage in addition to financial leverage. Debt dynamics are also different. While Goldstein *et al.* (2001) consider that debt only increases when the market value of assets goes up to a level where the firm wants to restore its optimal capital level, in this paper debt is continuously sold at market price meanining that net borrowing contribution to the FCFE is lower whenever the firm is performing poorly.[2] This debt dynamics has already been assumed by Ericsson and Reneby (2003) in a very similar model.[3] The estimation procedure is nevertheless very different. Though the project value in their paper derives its value from firm fundamentals (earnings before taxes in their case), this is not relevant in their estimation procedure. As a result, their asset value estimates are not compatible with observed fundamentals for the estimated model parameters. In addition, while Ericsson and Reneby (2003) use the model for bond pricing, the objective of this study is to measure the evolution of the equity risk premium implied by stock prices.

Other closely related article is the one from Eisdorfer *et al.* (forthcoming). These authors build an equity valuation based on gross profits (i.e. sales minus the cost of goods sold) that also considers operating and financial leverage effects on equity valuation. Their objective is nevertheless very different from mine. While I am trying to reverse-engineer the implied equity risk premium, Eisdorfer *et al.* (forthcoming) uses the CAPM model to price stocks and identify equity misvaluations arising from model misspecification. They also consider a more complex debt structure and thus they must restrict themselves to numerical solutions. In contrast, I assume a simple debt structure that allows me to find closed-form solutions.

This article aims at contributing to the contingent claims litetature in two ways. First, and ignoring the long formulas, this article proposes a model that is as simple to apply as the traditional single-stage FCFE model. The model can even be easily put on Microsoft Excel without requiring any VBA code. Though simple, the model accounts for a number of issues that are left outside the usual textbook model (default risk, operating and financial leverage effects). Second, the model is applied to S&P500 stocks at an aggregate level. This decomposition may help equity analysts judging whether stock prices are fairly priced, macroprudential authorities evaluating the risks

2. The roll-over process of the initial stock of debt is nevertheless not taken into account. See He and Xiong (2012) on this regard.

3. The model in this paper differs from theirs only on the state variable definition, the addition of operating leverage and the division of debt between interest-bearing and non-interest bearing.

to the financial system and monetary policy authorities understanding the impact of unconventional monetary policy on asset prices. For the best of my knowledge, this paper is the first to use a contingent claims model to extract the equity risk premium implicit in stock prices. Unfortunately, alike other studies that try to reverse-engineer equity valuation models, conclusions depend on how growth expectations are set.

The model

The FCFE model of equity valuation is one of the most popular among equity analysts. FCFE is a measure of how much cash is available to shareholders after all expenses, investment and net borrowing is taken into account. Firms can distribute this as dividends, buy back stocks or do nothing leading to an increase in cash accounts. Negative FCFE means that the firm has either to decrease its cash reserves, sell own shares held in its portfolio or issue additional equity to finance its activities. In contrast to dividend discouting, FCFE-based valuation models recognize that firms can also compensate their shareholders by repurchasing stocks, something that has become increasingly popular in the last decades. Taking the cash flow statement as the starting point to compute the FCFE we have that

$$FCFE_t = CFO_t + CFI_t + d_t, \tag{2}$$

where CFO_t refers to the cash flow from operations, CFI_t is the cash flow from investment and d_t corresponds to net borrowing. CFO comprises all cash flow the firm receives from its regular business activities. This includes all cash flow received from customers net of all expenditures with suppliers, fixed costs, corporate taxes and interest expenses. CFO is generally positive, though during recessions it may become negative, even for firms not in financial distress. In contrast to CFO, CFI is usually negative as it comprises investments in long-term assets such as property, plant and equipment (PP&E) and long term investments in other companies. However, it can also be positive when a firm sells its investments. Net borrowing is very irregular, but it tends to be positive over time following firm growth. As explained in the introduction, in the single-stage FCFE model, this is assumed to follow an infinite horizon discrete time deterministic trend process. In this article it is considered instead that $FCFE_t$ is a continuous time stochastic process with a finite horizon. This difference will turn the model significantly more complex but it will also allow us to better take into account the effect of business risk, default risk, operating leverage and financial leverage on the value of future FCFE.

Before presenting the free cash flow to equity dynamics, for reasons that will become clear soon, consider adding and subtracting in equation (2) fixed costs, q_t, and after-tax interest expense, which is hereafter presented as the product of the firm after-tax coupon rate c and total liabilities L_t:

$$FCFE_t = (CFO_t + CFI_t + q_t + cL_t) - q_t - cL_t + d_t. \tag{3}$$

The first term in brackets will be hereafter denoted as δ_t and assumed to follow a geometric Brownian motion with drift μ_δ and volatility σ:

$$\frac{d\delta_t}{\delta_t} = \mu_\delta dt + \sigma dW_t^{\mathbb{P}}. \tag{4}$$

The geometric Brownian motion is the same stochastic process Black and Scholes (1973) used to model stock prices. In this case it states the idea that in each moment in time the continuous compounding growth rate of our state variable δ_t follows a normal distribution with mean $\mu_\delta \Delta t$ and variance $\sigma^2 \Delta t$. This leads to a highly persistent process, which cannot take negative values.[4] For positive μ_δ and σ, the longer the time interval the higher is the expected value of our state variable and the uncertainty around its value.

Fixed costs, q_t, and nominal debt, L_t, are assumed to grow deterministically $\alpha q_t \Delta t$ and $\alpha L_t \Delta t$, respectively:

$$dq_t = \alpha q_t dt \tag{5}$$

and

$$dL_t = \alpha L_t dt. \tag{6}$$

It is further considered that nominal debt L_t is composed by a non-interest bearing component, L_t^{NonInt}, and an interest bearing component, L_t^{Int}. Each of these components follows an ordinary differential equation similar to the one given in equation (6). As a result, both components are a constant fraction of L_t. The fraction of non-interest bearing debt is denoted by φ. The owner of the interest-bearing component earns a coupon payment equal to $c^{Int} L_t^{Int}$. Since both components are a constant fraction of L_t, we have that the coupon rate on total liabilities, c is constant and equal to $(1-\varphi)\, c^{Int}$. For simplicity, it is considered that all initial debt and all new debt issues are perpetual. Non-interest-bearing debt is issued at nominal value, while interest bearing debt is issued at market value. The latter implies that the total cash inflow from new debt issues, d_t, is a function of the firm financial position at each moment in time. The lower the probability of the firm defaulting the higher the amount of cash flow it receives for the same level of additional nominal debt. Figure 1 (Panel A) shows examples of different δ_t paths along with total costs with coupon payments and fixed costs. Cash inflows arising from new debt issues under each δ_t path are presented in Figure 1 (Panel B).

4. One of the objectives of adding fixed costs to CFO is to address this problem. The other is to account for operating leverage.

In the traditional single-stage FCFE model, FCFE never takes negative values as it is assumed to grow at a constant rate up to infinity. In this model, however, δ_t may become less than $q_t + cL_t - d_t$ implying a negative $FCFE_t$. This is the case of the red path in Figure 1 (Panel C). Whenever the FCFE is negative, shareholders must decide whether they are willing to inject capital in the firm. They will do it until time τ, the first time δ_t hits a lower boundary $\overline{\delta}_t$, which is determined by solving the below equation:

$$\left.\frac{\partial E}{\partial \delta}\right|_{\delta=\overline{\delta}} = 0. \tag{7}$$

The above equation is known in the optimal stopping time literature as the smooth pasting condition. The intuition behind this is that shareholders are willing to inject capital as long as the equity value after the capital increase is higher than the amount of cash flow they inject. q_t and cL_t are crucial in shareholders default decision. Everything else equal, the higher the fixed costs the firm runs in its productive process (i.e. the higher its operating leverage) and its financial duties (i.e. its financial leverage) the earlier shareholders will give up the firm. It is important to emphasize that even if shareholders are liquidity constrained, in a world with no information problems and restrictions to capital movements, as long as the market value of equity after the capital increase is above the capital increase there will always be a price at which the firm will be able to raise capital. This occurs because, no matter the consequences in terms of dilution, it is always better for shareholders to raise capital than to lose the firm and receive nothing. The default barrier in our simulation exercise is presented in Figure 1 (Panel A) along with potential δ_t trajectories. Similar to L_t and q_t, $\overline{\delta}_t$ grows at rate α.

Whenever the barrier is hit, the firm is closed and distress costs are incurred. These correspond to legal costs and value destruction caused by fire sales and loss of intangible value. In this case the firm stakeholders receive βA_τ, where A_τ corresponds to the discounted present value of all future δ_t up to infinity. Mathematically,

$$A_\tau = \frac{\overline{\delta}_\tau}{r + \overline{m}\sigma - \mu_\delta}, \tag{8}$$

where r is the after-tax risk-free interest rate and $\overline{m}$ is the market price of risk (i.e. the amount of return demanded by investors by unit of risk). $\overline{m}$ can be interpreted as the project Sharpe ratio. The best way to understand this is to think that the firm continuously holds a project that generates δ_t up to infinity and whose value, A_t, corresponds to the perpetual sum of all future δ_t.[5] If δ_t becomes unsatisfactory the firm is closed and the project is sold to a

5. By applying Itô's lemma to the asset function it is possible to derive the dynamics of this fictive security. Since the market price of risk is assumed to be constant we have that $\sigma_A = \sigma$.

competitor firm. The project is infinite-lived but the firm is not. The β accounts for the fact that the firm stakeholders only receive a share of the project value when this occurs. The usual pecking order implies that shareholders only receive something if that share, i.e. βA_τ, is higher than nominal debt L_τ. For simplicity, it is assumed that β is sufficiently low so that shareholders receive nothing in case of liquidation. β affects equity value through the cash inflow, d_t, the firm receives when it issues new debt. The higher is β, the more debt holders recover after default, and thus the higher is the capital inflow whenever the firm issues new debt. β is thus a relevant parameter for equity valuation in this model.

For valuing this firm's stock, it is assumed the existence of a unique probability measure by which the discounted value of δ_t becomes a martingale.[6] Equity can then be valued as the discounted sum of all future after-tax free cash flows up to the moment the firm is closed plus its current after-tax cash position. Substantial cash holdings are a signal of potential dividends and stock buybacks. For this reason, cash holdings are very relevant for a shareholder that takes a "control" perspective over the firm.[7] Equity value in this model is obtained solving the below expression:

$$E_0 = \left(1-\bar{t}\right)\left(Cash_0 + E^{\mathbb{Q}}\left[\int_0^{+\infty} e^{-rs}(\delta_s - q_s - cL_s + d_s)\,1_{\{\tau>s\}}ds|\mathcal{F}_0\right]\right), \tag{9}$$

where $\bar{t}$ is interpreted as a weighted average of the expected dividend and capital gains tax rates and the term within the integral corresponds to the sum of all future FCFE until firm liquidation.[8] The expected value of the discounted sum of all future $\delta_s - q_s - cL_s$ is standard in the contingent claims pricing literature. For the sum of all future d_s we have to decompose it between cash inflow from non-interest-bearing and interest-bearing debt. The first expectation is also standard in the literature. For interest-bearing debt, which is sold at market value, it is assumed that the value of all future cash

6. A martingale is a stochastic process where the expected value of the next observation in the process equals the previous one. See Björk (2009) for a discussion on the technical conditions required for the existence of this unique probability measure.

7. Ideally, equity value should correspond to the discounted sum of all future amounts that the firm intends to distribute either through dividends or sharebuybacks. The firm payout depends on FCFE but also on its cash holdings. It is reasonable to think that when cash holdings are considered low, management might decide to retain part of the FCFE. The opposite may occur when cash holdings are high. In this case, management can even decide to distribute more than its current FCFE. The proper treatment of cash holdings would turn the model significantly more complex. Treating initial cash holdings as an amount that is immediately available for distribution is a simplified way of addressing this issue.

8. Notice that only these taxes need to be taken into account in equation (9) since the state variable already accounts for corporate level taxation.

inflows must equal the value of all coupons that accrue to the new debt issues plus their share on the recovered value after firm liquidation.[9] Figure 1 (Panel D) illustrates possible equity trajectories in the context of our simulation exercise.

The reader less familiar with the idea of risk neutral pricing may find strange discounting the future FCFE at the risk-free rate. However, under this framework investors compensation for taking risk is taken into account by changing the probabilities of the different outcomes instead of demanding an higher discount rate. A very common misconception is that risk neutral pricing implies zero risk premiums. This is however not true. Notice that, in contrast to the traditional Black-Scholes-Merton framework, where the existence of a replicating portfolio leads derivatives to earn a zero risk premium, in this case it is impossible to form such portfolio because the underlying asset is not traded. It is however possible to show that one can form an instantaneously risk-free portfolio given any two traded contingent claims. Similar to the derivative contract in the Black-Scholes model, the second contingent claims asset is superfluous because its dynamics can be replicated using the first contingent claim contract and a risk-free bond. In other words, the existence of a market for a contingent claim (e.g. equity) guarantees that the price of all other contingent claims (e.g. CDS) are uniquely determined by this benchmark (see Björk (2009)). The second contingent claim depends on the market price of risk because the first contingent claim also depends on the market price of risk. The possibility of bulding a risk-free portfolio using these two claims does not eliminate this dependence. The risk-neutral pricing framework has two important advantages over standard valuation methods that discount cash flows under the physical measure. First, under standard methods the discount rate is often constant and thus independent of the firm performance. However, it makes sense to set higher discount rates whenever the underlying investment becomes riskier. As it is shown in Figure 1 (Panel E) and better explained in the end of this section, besides a constant market price of risk, $\overline{m}$, the cost of equity in this model increases (decreases) when the firm performs poorly (well). Second, the risk-neutral approach allow us to price all contracts that are contingent on the firm's business without having to compute their specific discount rate. This can be very appealing whenever one wants to extend the methodology to other contingent claims such CDS contracts.

A CDS is a contract by which its seller agrees to compensate the buyer in case of a credit event. In return, as long as the underlying entity does not default, the CDS buyer makes a series of payments to the seller, the CDS spread. This is the coupon value that turns the expected value of future credit

9. See Appendix for more information on how the present value of all future debt issues is computed.

losses equal to the expected value of these payments. Mathematically, this value can be found by solving the below equation:

$$E^{\mathbb{Q}}\left[cds\int\limits_{0}^{t^{cds}} e^{-rs}1_{\{\tau>s\}}ds\,|\mathcal{F}_0\right] = E^{\mathbb{Q}}\left[e^{-r\tau}1_{\tau<t^{cds}}|\mathcal{F}_0\right] - E^{\mathbb{Q}}\left[e^{-r\tau}Rec_{\tau}\right], \tag{10}$$

where t^{cds} is the CDS maturity and $E^{\mathbb{Q}}\left[e^{-r\tau}Rec_{\tau}\right]$ stands for the discounted expected recovery rate. The recovery rate associated with the CDS contract depends on the nominal value of the debt class insured and on the amount of senior liabilities. The higher are senior liabilities and the debt class insured the lower is the recovery rate associated with the CDS contract. Depending on the relation with the default barrier one may have zero recovery (when the recovered amount is lower than senior liabilities), total recovery (when the recovered amount is higher than senior liabilities plus the nominal liabilities associated with the debt class insured) or partial recovery. Mathematically,

$$E^{\mathbb{Q}}\left[e^{-r\tau}Rec_{\tau}\right] = \begin{cases} 0, \beta\overline{v}_0 \leq X \\ \left(\frac{\beta\overline{v}_0 - X}{L^*}\right)E^{\mathbb{Q}}\left[e^{-r\tau}1_{\tau<t^{cds}}|\mathcal{F}_0\right], X < \beta\overline{v}_0 \leq X + L^*, \\ E^{\mathbb{Q}}\left[e^{-r\tau}1_{\tau<t^{cds}}|\mathcal{F}_0\right], \beta\overline{v}_0 > X + L^* \end{cases} \tag{11}$$

where $L^{\star}$ is the nominal value of the debt class insured, X is the amount of liabilities senior to the debt class insured, which is assumed to grow at the same rate as L, and $E^{\mathbb{Q}}\left[e^{-r\tau}1_{\tau<t^{cds}}|\mathcal{F}_0\right]$ is the value of a claim that pays unity whenever the firm is liquidated. The computation of this expectation is standard in the literature. Figure 1 (Panel E) illustrates CDS spreads trajectories in the context of our simulation exercise.

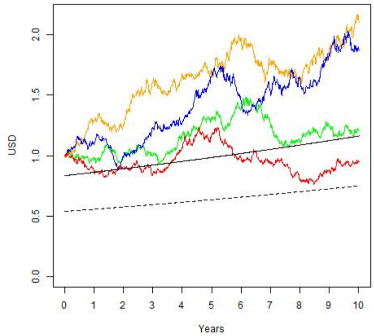
Equation (9) can be used for equity valuation whenever one is able to provide estimates on all model inputs. Alternatively, one can use observed equity prices to extract the market price of risk $\overline{m}$ implied by stock prices. This can then be used to compute the equity risk premium and the cost of equity. The latter corresponds to the drift of the equity process under the physical measure. This is given by

$$\mu_{E_t} = r + \overline{m}\sigma_{E_t}, \tag{12}$$

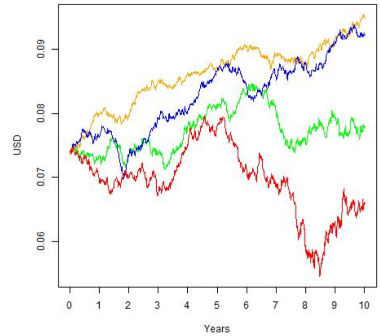
where σ_{E_t} refers to equity return volatility. This can be obtained from the application of Itô's lemma to the equity function

$$\sigma_{E_t} = \frac{\partial E}{\partial \delta_t}\frac{\delta_t}{E_t}\sigma. \tag{13}$$

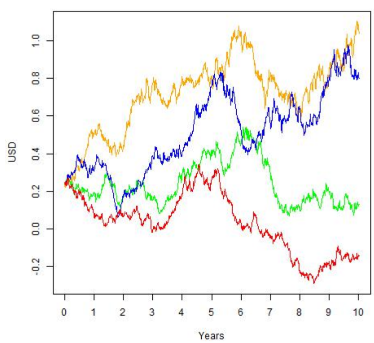
Though business risk, σ, is constant, the existence of fixed operating and financial costs lead the derivative of equity regarding δ and the ratio between δ_t and E_t to be not constant. This leads to a stochastic cost of equity that can be well seen in our simulation exercise presented in Figure 1 (Panel F).



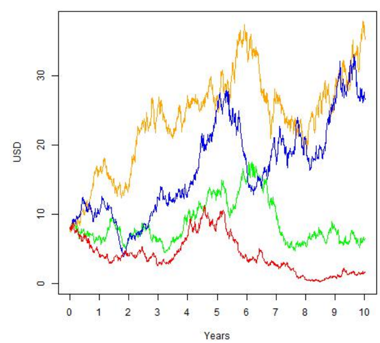
(A) Examples of different δ_t paths.



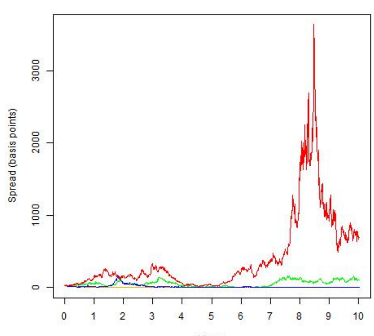
(B) Cash inflow from new debt issues.



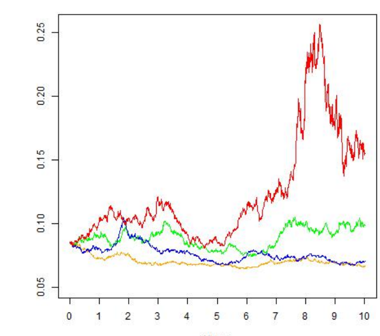
(C) FCFE



(D) Equity value



(E) CDS (5-years)

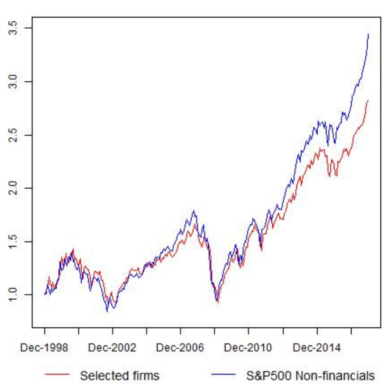


(F) Cost of equity

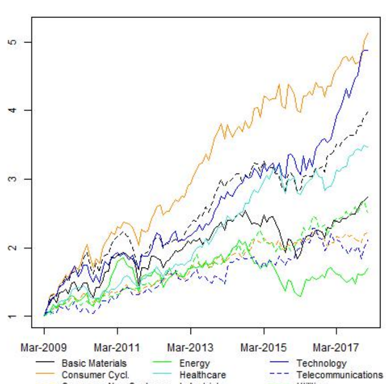
FIGURE 1: Simulation exercise. $\delta_0 = 1$, $r = \mu_\delta = \alpha = 0.033$, $\sigma = 0.106$, $q_0 = 0.79$, $c = 0.016$, $L_0 = 2.65$, $\overline{m} = 0.133$, $\beta = 0.049$, $\overline{t} = 0.15$, $Cash = 0.23$, $X = 1.64$ and $\varphi = 0.57$. The values used are normalized values based on December 1998 calibration. In Panel A, the continuous think black line corresponds to the sum of interest and fixed costs and the dashed black line is the default barrier.

Data and calibration

This section presents the data and calibration procedure used in this study. All data is collected from Thomson Reuters for the period between December 1998 and December 2017. Accounting data is collected with annual frequency, while market data is collected with monthly frequency. The initial dataset corresponds to 406 non-financial firms composing the S&P500 in December 2017. This was subsequently restricted to 205 firms in order to include only those firms for which all the required data is available for the whole period. The large majority of the firms excluded did not exist or were not listed in December 1998. Except for technology, basic materials and telecommunications, sampled firms represent more than 60% of each sector market capitalization. This figure falls to approximately 40% for the technology and basic materials sectors. The telecommunications sector is not represented in the sample. Figure 2 (Panel A) compares the evolution of the market capitalization for these firms with an index based on the initial sample of firms controlling for entrances and exits. Figure 2 (Panel B) shows similar indices per sector of activity, but starting in March 2009, when market indices reached their bottom. Despite the two series following a similar trajectory, it is clear that firms on our sample have had an increase in market capitalization below others. Rather than a sector underrepresentation problem, this seems related with the predominance of mature firms in the sample. A point can obviously be made that the selected sample of firms does not totally capture the recent increase in the $S\&P500$. Though true, the fact that our sample of firms is constant across time better allow us to study what is going on.



(A) Market capitalization (1998-2017) normalized in December 1998. Comparison between sample and initial aggregate.



(B) Market capitalization per sector of activity (2009-2017) normalized in March 2009.

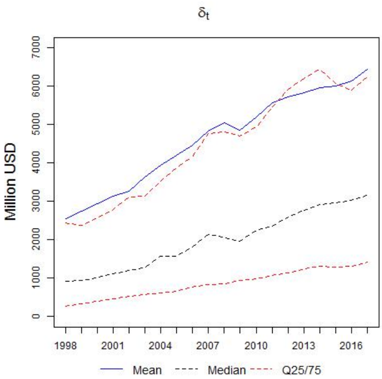
FIGURE 2: Market capitalization.

The model presented in the previous section has 14 inputs, notably, the sum of the cash flow from operations, the cash flow from investment activities, fixed costs and after-tax interest expenses (δ_0), fixed costs (q_0), short term financial assets ($Cash_0$), total liabilities (L_0), senior liabilities (X), the share of non-interest-bearing liabilities (φ), after-tax coupon rate on total liabilities (c), dividend and capital gains tax rate ($\bar{t}$), the after-tax risk free rate (r), expected growth rate of debt (α), expected growth rate of the state variable (μ_δ), business risk (σ), the amount of return demanded by investors by unit of risk ($\overline{m}$) and a recovery rate-related parameter (β). δ_0, q_0, $Cash_0$, L_0, X_0, φ_0 and c are readily available from financial documentation and presented in Figure 3. δ_0 was computed summing cash flow from operations, cash flow from investment activities (smoothed), SG&A and after tax interest expense.[10] Similar to Eisdorfer *et al.* (forthcoming), SG&A, which includes all costs that cannot be tied directly to the firm's output, is thus used as proxy for firms' fixed costs, q_0. SG&A represents on average 76% of our state variable. $Cash_0$ corresponds to the cash account plus other short term financial assets. L_0 corresponds to total non-equity liabilities excluding minority interests. X_0 equals L_0 minus long-term debt. φ was set as 57%, which corresponds to 1 minus the ratio of total debt outstanding to total liabilities in Reuters. Finally, c was computed as interest expense divided by total nominal liabilities and multiplied by 1 minus the corporate tax rate, which was assumed to be 20%.[11] δ_0, q_0, $Cash_0$, L_0 and X_0 correspond to the sum of all individual firms observations. c is the weighted average based on each firm end-of-month market capitalization. r was obtained multiplying the yield on 30-year U.S. Treasury bonds by 1 minus the interest income tax rate, which was assumed to equal 35%. $\bar{t}$ was set at 15%. α was assumed to be equal to μ_δ in order to keep the expected value of the leverage ratio constant across the firm's life.

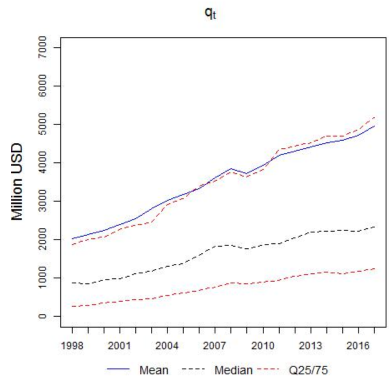
Two different assumptions are considered regarding μ_δ. First, it was assumed that corporate long-term growth rate equals the risk free rate (i.e. $\mu_\delta = r$). This assumption is very common in equity valuation. The idea behind is that one day the firm will stop over or underperforming the economy and converge to its long-term nominal rate of growth. The relationship between

10. There is considerable variation in capital expenditure. For this reason, Eisdorfer *et al.* (forthcoming) assumes that capex expenditure at each moment in time equals the three-year average capex-to-sales ratio times firm sales at each moment in time. In this article, I started by computing each year contribution (% share) to the accumulated state variable (between 1999 and 2017). The cumulative cash flow from investment activities was then multiplied by this value. This procedure avoids using other firms data, which can be a problem whenever there is significant capex variation within the industry. The main drawback of this procedure is that it is prone to backwards revisions of the model output. This should be however minor for a sufficiently large number of years.

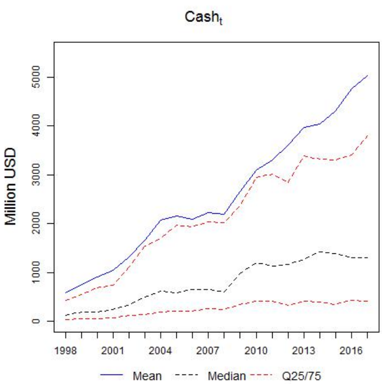
11. The corporate tax rate is not very important in this model because the CFO is computed after tax. Changing the corporate tax rate assumption will only slightly affect the firm's financial leverage and thus the optimal default barrier.



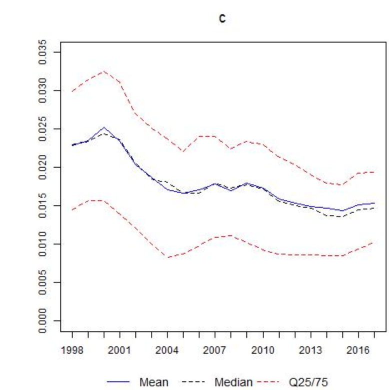
(A) State variable



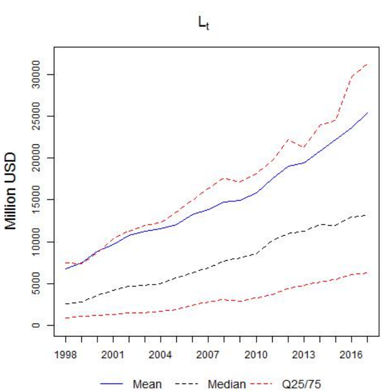
(B) Fixed costs



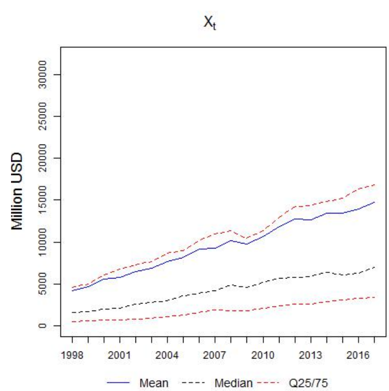
(C) Cash and equivalents



(D) Coupon rate



(E) Total debt



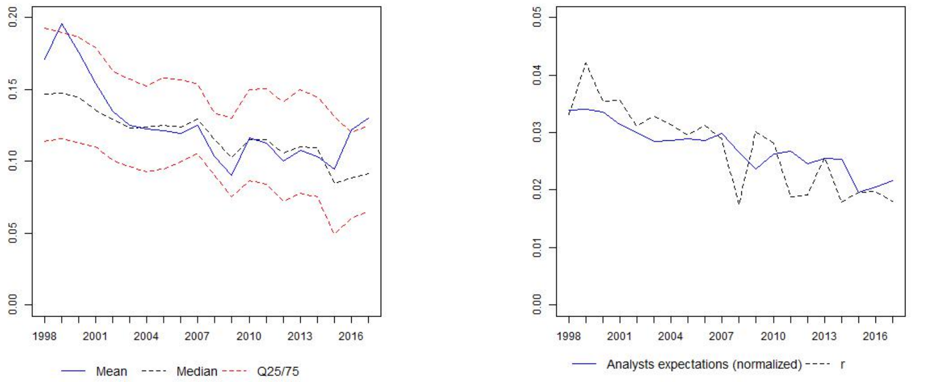
(F) Senior debt

FIGURE 3: Firm fundamentals.

economic growth and risk-free rates has solid theoretical underpinnings. I will skip reviewing this literature here. Intuitively, it is reasonable to think that, everything else equal, decreases in long-term risk-free rates signal that investors are preferring to invest in risk-free bonds rather than investing in shares, bonds or real assets. The results obtained were compared with the ones that come out from assuming that μ_δ is a multiple of analysts 3 to 5-years earnings forecasts (compounded growth rate). These were taken from Thomson Reuters I/B/E/S database and are presented in Figure 4 (Panel A). Studies on analysts' capacity to correctly forecast corporate growth have generated mixed results. For the sample of firms considered, a moderate correlation (42%) is found between the compounded annual average growth rate of analysts' forecasts and the compounded annual average growth rate of our state variable between 1999 and 2017. More interestingly, a correlation of 89% is found between median analysts' forecasts and 30-year U.S. Treasury bonds during the same period (Figure 4 Panel B). Despite this high correlation being the result of the two series following basically the same trend (the correlation almost disappears when the two series are detrended), it suggests that analysts' forecasts can be used as an alternative to long-term nominal rates. The fact that these forecasts reflect analysts' momentum on firm fundamentals is useful to understand what is leading stock markets. In line with the literature that points out that analysts' forecasts are generally too optimistic, the average annual growth rate of analysts' forecasts is found to be approximately 6 percentage points above the annual growth rate of our state variable. Analysts' forecasts are also very high to be thought as sustainable long-term growth rates. For these reasons the obtained figures were scaled down by multiplying by the mean ratio between r and analysts' growth forecasts.[12] The median value was then chosen as proxy for long term growth expectations. The median value was preferred to the weighted mean because it is less sensitive to abrupt changes in analysts' forecasts on some very large firms. This is particulary relevant given the high sensitivity of equity value to this parameter in this model.

In line with the model assumptions, σ, which captures business risk, was considered to be constant across the whole estimation period. As it is clear from Figure A.1 in the Appendix, this does not imply constant equity volatility. Each firm σ was estimated through a robust linear regression of the logs difference of the state variable δ_t on a constant. Figure 5 shows an histogram of these estimates. Approximately 40% of our σ estimates lie between 8% and 15%. The 10th and 90th percentile of the distribution are 5.2% and 25.1%, respectively. Since the exercise in this article was carried at

12. The use of a multiple of analysts' forecasts is also done in the well-known Yardeni model (Yardeni (2003)). This multiple is not computed in the same way, though.



(A) Analysts' growth forecasts (end-of-year).

(B) Normalized median analysts' forecasts and 30-year U.S. bond yields (end-of-year).

FIGURE 4: Long-term growth expectations.

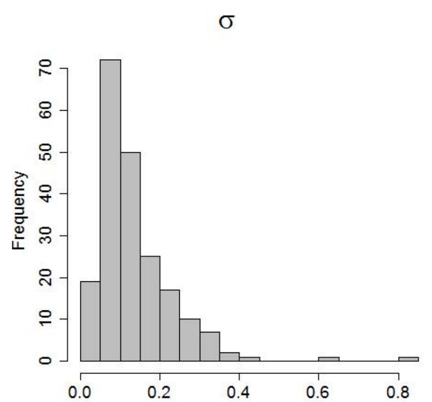


FIGURE 5: Histogram of σ estimates.

the aggregate level, σ was set as the median of individual volatility estimates (i.e. 0.106).

Finally, $\overline{m}$ and β are estimated by solving a system of equations where $\overline{m}$ and β are chosen so that equity value in the model matches the observed market capitalization and CDS spreads. A weighted average of the CDS spreads (5-years) of 62 firms is used (Figure 6).[13] Given the lack of CDS data of

13. This procedure was carried with monthly frequency between December 1998 and December 2017. Monthly accounting figures were linearly interpolated from annual figures.

good quality for the period before 2009, in this period $\overline{m}$ and β were estimated assuming a recovery rate of 0.23. This corresponds to the average recovery rate obtained during our exercise for the period after 2009.

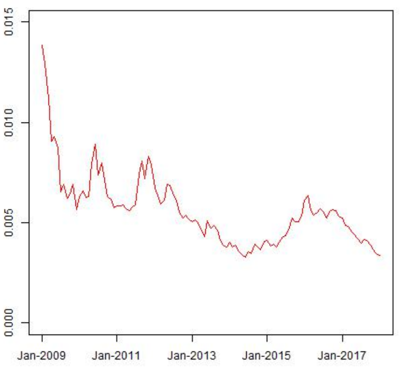


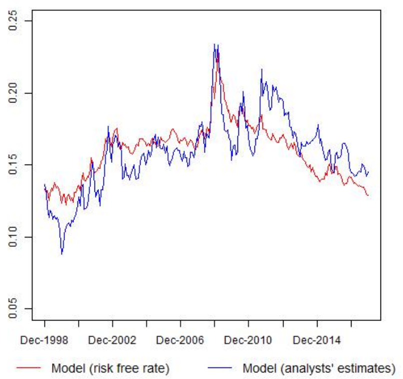
FIGURE 6: Credit default swap spreads (5-years).

Results

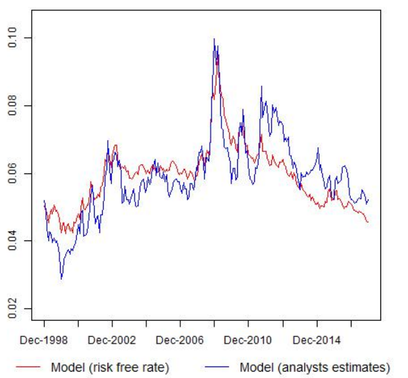
Figure 7 shows the market price of risk (Panel A) and the equity risk premium (Panel B) obtained assuming growth expectations based on the risk-free rate and on long-term analysts' forecasts, respectively. The two panels are very similar. The market price of risk and the equity risk premium are not a multiple of each other only due to equity volatility, which is not constant across time (see Figure A.1 (Panel A) in Appendix).[14] A mean equity risk premium of approximately 5.9% is observed in both cases. The two series also have a similar pattern, marked by very low values in the beginning and in the end of the estimation interval and very high values during the financial crisis. Currently, the equity risk premium is in a downward trend reaching 4.6% in the end of 2017 when the risk free rate is used and 5.2% when analysts' forecasts are used. It is interesting to note that while in the first case the equity risk premium is very close to the minimum of the series, in the second case it is somewhat closer to the average. The equity risk premium is nevertheless significantly more volatile in this second case because the model

14. Though business risk measured by the state variable volatility is constant, financial and operating leverage lead to stochastic volatility. In our case, equity volatility ranges from 0.33 to 0.42. This range is very narrow when compared with the one that is empirically estimated (see Figure A.1 (Panel B) in Appendix).

is very sensitive to differences between growth expectations and the risk-free rate. Given that the differential between the risk-free rate and analysts' normalized growth forecasts tend to be positive before 2010 and negative afterwards, these differences lead the equity risk premium resulting from assuming growth expectations based on the risk-free rate to be almost always above those based on long-term analysts' forecasts until 2010 and below subsequently.[15]



(A) Market price of risk.

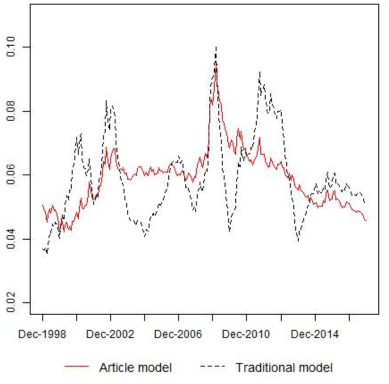


(B) Equity risk premium.

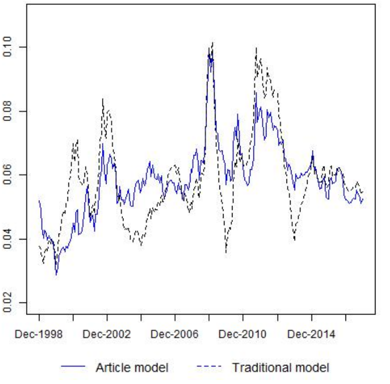
FIGURE 7: Model implied market price of risk and equity risk premium.

The results obtained with the model presented in this article are not materially different from those that come out from the traditional single-stage FCFE model (Figure 8). Adjusting for taxes and cash holdings an implied equity risk premium of 5.9% is also found in this case. The two series have nevertheless a correlation that is far from perfect (56% when growth expectations equal the risk-free rate and 74% when growth expectations are proxied by analysts' forecasts). This is largely the result of the series that comes out from the traditional single-stage FCFE model being significantly more volatile under both expected growth assumptions. The very significant increases in the equity risk premium observed in March 2001, September 2002 and September 2011 are good examples of this. These spikes are observed under both growth rate assumptions in the case of the traditional FCFE model. However, when the structural model is applied these spikes are very contained, especially when growth expectations equal the risk-free rate.

15. It is also interesting to note that when growth expectations are equal to the risk free rate, despite some small spikes being observed during the European sovereign debt crisis, the implied equity risk premium is very far from the levels observed during the peak of the financial crisis. In contrast, when growth expectations are based on financial analysts forecasts, the equity risk premium jumps significantly in the second half of 2010 and 2011.



(A) μ_δ based on the long-term risk-free rate.



(B) μ_δ based on analysts' forecasts.

FIGURE 8: Equity risk premium. Comparison with the single-stage traditional FCFE model.

Concluding remarks

This article derives the equity risk premium implicit in S&P500 stock prices using a single-stage FCFE-based structural model. An aggregate perpective was followed. In line with literature and historical observation, a mean equity risk premium of approximately 5.9% is found for the period between 1999 and 2017. Independently of using the risk free rate or a multiple of analysts' forecasts the equity risk premium is found to be currently on a downward trend. The level observed in December 2017 is nevertheless different depending on how growth expectations are set. While in the first case, the equity risk premium is found to be 4.6%, very close to the minimum of the series, in the second case it is found to be 5.2%, somewhat closer to average. This difference is justified by the recent apparent decoupling of normalized analysts' forecasts from 30-year U.S. bond yields. This decoupling can be interpreted as a signal of analysts optimism on quoted firms future performance. However, it can also be related with the current abnormally low level of long-term interest rates given the U.S. economy fundamentals.

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Appendix

The discounted sum of all future d_s can be decomposed between cash inflow from non-interest-bearing and interest-bearing debt. Since non-interest-bearing debt is sold at nominal value we have that:

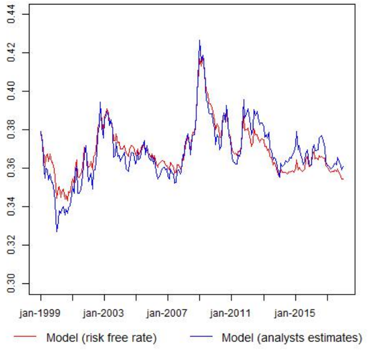
$$E^{\mathbb{Q}}\left[\int_0^{+\infty} e^{-rs} d_s^{NonInt} 1_{\{\tau>s\}} ds|\mathcal{F}_0\right] = E^{\mathbb{Q}}\left[\int_0^{+\infty} e^{-rs} \mu_\delta \varphi L_s 1_{\{\tau>s\}} ds|\mathcal{F}_0\right]. \tag{A.1}$$

The solution of equation (A.1) is standard in the literature. For interest-bearing debt, it is assumed that the value of all future cash inflows must equal the value of all coupons that accrue to the new debt issues plus their share on the recovered value after firm liquidation. Mathematically,

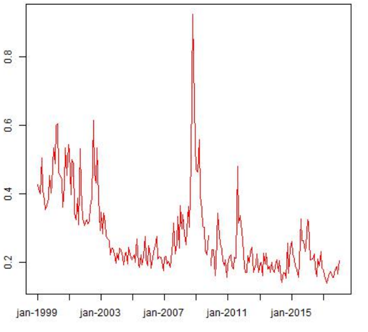
$$E^{\mathbb{Q}}\left[\int_0^{+\infty} e^{-rs} d_s^{Int} 1_{\{\tau>s\}} ds|\mathcal{F}_0\right] = E^{\mathbb{Q}}\left[\int_0^{+\infty} e^{-rs} \left(cL_s - cL_0\right) 1_{\{\tau>s\}} ds|\mathcal{F}_0\right] + \left(1-\varphi\right)\beta E^{\mathbb{Q}}\left[e^{-r\tau}\left(\overline{v}_\tau - \overline{v}_0\right)|\mathcal{F}_0\right], \tag{A.2}$$

where $\overline{v}_0$ is the project value that leads the firm to default at time 0. The solution to equation (A.2) is standard in the literature.

Figure A.1 shows model implied equity volatility as given by equation 13 and empirical volatility estimated as the annualized monthly standard deviation of daily returns.



(A) Model implied volatility.



(B) Empirical volatility.

FIGURE A.1: Equity volatility.